Chapter 2 – **Standards for Measurement**

2.1 Scientific Notation

Numbers are integral to understanding and communicating science. When we describe items or phenomena in science, we can do so qualitatively (e.g. the apples are red) or quantitatively (there are 3 apples). Clearly, in the latter case, you must use numbers. One problem with numbers can be size because they are sometimes very large or very small. For example, if you have a coffee cup filled with strawberries, determining how many are there and communicating that is simple; you simply count them. On the other hand, if you filled the back of a pickup truck with strawberries the problem is more difficult. You could count the strawberries and write the number down, but that is generally impractical for numbers this large. In addition, writing out large numbers longhand can have problems including getting the number of digits wrong (e.g. mixing up 10000000000 and 100000000000 is easy) and getting the accuracy of the number wrong.

The method of expressing very large and very small numbers is called scientific notation. In it, numbers are expressed as powers of ten. For example, 1200 becomes 1.2×10^3 (i.e. $1.2 \times 1000 = 1{,}200$). For a number like 1,200, this may not seem like it is helpful, but, as well see shortly, there are many times that writing one thousand two hundred this way is the best way to express a number. Where it is clearer that scientific notation is helpful is for a number like $1{,}000{,}000{,}000{,}000 = 1 \times 10^{12}$ or $0.0000000027 = 2.7 \times 10^{-9}$. Here the advantage is that if you have to count zeros, you've opened yourself up to an easy error.

To write a number in scientific notation, you take the number and move the decimal point so that it resides immediately to the left of the first digit and count the number of places you have to move the decimal point to generate the exponent on 10. For example, consider 18,000. You move the decimal point 4 places to the left to place it between the 1 and 8 so it becomes 1.8×10^4. It works the same way for very small numbers: 0.00000018 becomes 1.8×10^{-7} because you move the decimal point 7 places to the right.

2.2 Measurement and Uncertainty

Broadly speaking, numbers may be broken into two categories exact and inexact. An exact number has no error, while an inexact number does have error. A classic example of an exact number is the dozen. It has exactly 12 items with no other value possible. Your waist size is a classic inexact number. For example, how accurately is it measured? To the nearest inch? Quarter inch? Etc. How tightly is the tape measure pulled? How accurate is the measuring device? There are other sources of error that make it impossible to know with 100% accuracy a waistline.

Scientific notation is a particularly good way of letting the reader know how accurate a particular number is. For example, if I told you my car had an MSRP of $25,000, chances are that you would understand me to mean the car cost about $25,000, not exactly $25,000. Thus, you would understand the 2 and 5 digits to have more meaning than the three trailing zeros, which really are just telling you the order of magnitude of the number. In this case, we would express $25,000 as \$$2.5 \times 10^4$ because the zeros are just place holders. In contrast, if I said I wrote a $100 check as a donation to a charity, you would probably assume the check was actually for $100 and that number would be written as 1.00×10^2 because the zeros not only give you the magnitude of the number, but also information on the actual value of the number.

In the laboratory, numbers are either exact (e.g. counting) in which all of the digits are significant and appear in the scientific notation. Thus, a ton is 2.000×10^3 pounds. Measurements always have error, however and so it is typically the case that numbers associated with measurements will not have all of their digits appear. Here, the number is expressed so that only the digits you can actually measure are in the part with the decimal point.

2.3 Significant Figures

Significant figures (digits) are those parts of a number that have physical meaning. All measured values have an upper limit to their accuracy and significant figures are how that is presented to the reader. Let's use my height as an example of this. If you used a ruler that had only 1' markings, you would say I was 6' tall because I would clearly be closer to 6' than either 5' or 7'. If your ruler had 1" markings, you could get a more accurate measurement of my height, now 5'8". If there were more gradient markings, you could see that I'm 5'8$\frac{1}{4}$". Having my height expressed in two units (feet & inches) is inconvenient, so let's convert to inches only. My height is then 68 inches and 68.25 inches, respectively. In the first case, since the ruler has only 1″ markings, we only know my height to ±1″ with certainty. We can see that it is closer to 68″ than either 67″ or 69″, but those are only estimates. Only when we have a more accurate ruler can we say with certainty that my height is 68.25″. Thus, the more accurate ruler allows us to write numbers with more significant digits in it.

A problem arises when we get to very large numbers, however. For example, if I ask how much a ship weighs, there is no way to put it on a scale. A US destroyer is listed as weighing 9300 tons (or 1,860,000 lbs). Going back to a few paragraphs, does the destroyer actually weigh exactly that amount? Obviously, it's very unlikely that it is accurate to the nearest pound. Just looking at the number, we know that the 1, 8, and 6 are significant, but we know nothing about the zeros. They could just be placeholders to tell you the ship weighs a little under two million pounds or it might be that the thousands digit is correct. There is no way to know just looking at the number to be certain. To be safe, we always use the smallest number of significant figures when discussing the accuracy of the number. Scientific notation gives us a convenient way of making sure the reader knows the accuracy of the number. If the ship's weight is accurate only to ±10,000 lbs, its weight is 1.86×10^6 lbs, but if it's accurate to the nearest 1,000 lbs, its weight is 1.860×10^6 lbs.

Pages 17 and 18 of your book has the rules for deciding if a digit is significant or not. Zeros are the problem number. All digits are significant except zeros whose only function is to tell you the magnitude of the number. Thus, in the case of 1,860,000 all four zeros are not significant because their function is to tell you the ship weights about a million pounds. For, 0.00032 the zeros are likewise not significant because they tell you how small the number is. In contrast, for 0.000302, the zero between 3 & 2 is significant because the number is between 0.000301 and 0.000303 and so is being measured. Finally, it is good practice to write out the "0" to the left of a decimal point when the number is smaller than 1.

2.4 Significant Figures in Calculations

When using numbers in calculations, the number of significant figures (SF) will play a part in the answer. There are two sets of rules regarding the number of SF in your answer depending on whether the operation is multiplication/division or addition/subtraction. We will look at these separately.

Multiplication/Division

When doing multiplication or division problems, you simply determine the number of significant digits in each number, perform the operation, and then round the answer to have the same number of SF as the starting number with the fewest SF. For example, 12 x 123 = 1476. When reporting the answer with the correct number of SF, "12" has two SF, "123" has 3 SF, so your answer would have only two SF. Thus, the correct result would be 1.5×10^3. In science courses, it is very important to avoid the temptation to write down whatever appears in your calculator's display.

There is one commonly accepted modification to this rule, which is to list one extra SF. In this case, the answer reported would be 1.48×10^3. The reason for this is that when we read the last digit from a measurement, the error is frequently about half of ±1 (i.e. When reading a value, the reading will be closer to the number recorded than to the next higher or lower value. E.g. if the reading is "1" the actual value is closer to "1" than to "2" or you would record it as a "2."). Thus, the answer is a bit more accurate than the formal rules of significant figures would suggest.

Addition/Subtraction

The rule here is quite different. For addition/subtraction, the SF are truncated at the significant digit in the starting number with the highest magnitude. This is probably confusing to read, but an example should be much clearer. Consider adding 150 to 2489:

2489 + 150 = 2639

Here, the zero is not significant, so you cut off the ones place in the sum, making the answer = 2640. Thus, the ones place is significant in 2489, but not 150, so you truncate at the tens place, which is significant in both numbers.

A quirk of addition and subtraction is that significant digits can be created or destroyed. The effect is real, but may be a bit unsettling at first. Consider adding 5 and 6. The answer 11, has 2 SF, while 5 and 6 each have one. The reverse process, subtracting 85 from 91, yields 6, so our starting 85 and 91 each has two SF, which is reduced to one SF in the answer.

Finally, when a problem contains both multiplication/division and addition/subtraction calculations, the rules are followed as the operation is performed. Thus, for (5+6)*8, the mathematical rules for doing the calculation would be do the addition first, then the multiplication would mean

(5+6)*8 = (11)*8 = 88

but "8" has the smaller number of SF, so the answer would round to 90 (or 9×10^1).

2.5 The Metric System

Although the English measuring system (e.g. feet, pounds, quarts) is familiar, as you know, it's also quite clunky. Converting between units, say inches ⇆ feet ⇆ yards ⇆ miles as you move from small amounts/distances to large is either a minor hassle or one that requires a calculator to get it correct. For example, how many miles is 2.3 inches? Almost no one could do this without a calculator or a sheet of paper and 10 minutes to work it out. Furthermore, all of the conversion factors are different; forcing you to remember tables of numbers to work.

For these reasons, the metric system was developed. It is almost universally both in science and economies (only Myanmar, Liberia, and the United States don't) because of its ease of use. You will need to know the basic units used in chemistry and several of the conversion factors:

distance = meters volume = liters mass = grams

For comparison, a meter is about 10% longer than a yard, a liter is about 5% larger than a quart, and a paperclip or dollar bill weighs about 1 gram. You don't have to memorize that sentence, but you will probably find doing so very helpful when working problems. Metric units are typically very unfamiliar to students, so when doing a calculation if the number is unreasonable it's difficult to know it. For example, is a gas tank volume of 60 liters a reasonable number? If you know a liter is a little larger than a quart, than 60 liters is a little more than 60 quarts or 15 gallons (4 quarts in a gallon), so the value is reasonable. It might not be exactly correct, but it isn't obviously wrong. In contrast, if you make a mistake entering a number in your calculator, say double hitting a number and so instead of multiplying by 220, you multiply by 2220, your answer will be ten times too large. In that case, you might wind up with a gas tank with a volume of 600 liters (or 150 gallons). Seeing that would tell you to go back and look for a mistake in your calculation.

The simplicity of the metric system comes from the conversion factors, which are all powers of ten. In the example in the first paragraph, the conversion factors are: 63,360 inches ⇆ 5280 feet

⇆ 1760 yards ⇆ 1 mile. In contrast, in the metric system, each unit is a factor of 10 larger or smaller than the unit below or above it, respectively, so you change units by just moving the decimal point. You must memorize the boldfaced conversion values and prefixes in Table 2.1 (p. 22 of your book). Furthermore, you must memorize one English to metric conversion factor each for length, mass, and volume. Some of the conversion factors may be somewhat familiar to you. For example "centi-" means 1/100. A century is 100 years, so by analogy there are 100 centimeters in 1 meter. Similarly, a decade is 10 years and a dekaliter (more commonly decaliter) is 10 liters.

An example to illustrate the advantage of metric units in conversion would be converting a short distance to a longer one. 3 inches is about 7.6 centimeters (cm). To go to a larger unit, 3 inches is 3/36 of a yard or 0.0833 yds, while 7.6 cm is 0.076 meters. As you can see, the math involved is much simpler for the metric units.

2.6 Dimensional Analysis: A Problem-Solving Method

This is one of the mo`st important topics in CHM 111. There is a great deal of math in chemistry courses and this is the method all chemists use both in classes and their laboratories. Dimensional analysis uses units to guide the person doing the calculation, rather than forcing one to memorize every formula and every modification to those formulae. It is sometimes called the factor-label method. It is important to remember that while in many circumstances it saves one the need to memorize a formula, that isn't always the case. We'll see an example of that and how you can identify the equations you must memorize.

Consider the following problem: How many nickels are in \$1.25? Chances are you took \$1 and divided by 5 to get the number of nickels in the dollar, then the same thing on the quarter, finally adding the values to get 25 nickels. That process is essentially dimensional analysis. Here is what that problem looks like using dimensional analysis:

$$\text{nickels} = (\$1.00)\left(\frac{100\text{ cents}}{\$1}\right)\left(\frac{1\text{ nickel}}{5\text{ cents}}\right) + (25\text{ cents})\left(\frac{1\text{ nickel}}{5\text{ cents}}\right) = 25\text{ nickels}$$

which we can simply to

$$\text{nickels} = (\$1.25)\left(\frac{100\text{ cents}}{\$1}\right)\left(\frac{1\text{ nickel}}{5\text{ cents}}\right) = 25\text{ nickels}$$

these equations are in the format you **must use** in chemistry courses. The advantage of this method is that you can use the same basic equation to figure out the amount of any currency for any dollar amount. You simply plug in how much money you have and what coin or paper money you want to convert it into. This generality makes dimensional analysis the easiest way to work nearly all math problems in chemistry. It works by successively cancelling out the units you have and replacing them with units either in the answer or which lead you to the answer.

Entry to CHM 111 requires a proficiency in basic algebra. One challenge with chemistry problems is that they are word problems, rather than problems from a math text. Dimensional analysis helps work around the structure of chemistry problems. Begin by identifying all of the information that **might** be used to solve the problem. Since these are math problems, write down all of the numbers with units as *givens*. In a typical problem, you will use every number in the solution. Next, identify what you are solving for, with units. Consider Advanced Example 2.16 on p. 28. "A 1.50-lb package contains how many grams of baking soda?"

Given: 1.50 lb_{BS} **Find**: $mass(g)_{BS}$

Note that the number written next to "Given" has all 3 SFs, the unit, and a subscript providing the identity of the material. For "Find," the property, unit, and material identifier are all provided. At this point, it is natural to wonder why one should include the identifiers. The answer is that as problems become more sophisticated, you may have multiple materials and 1 gram of water is different from 1 gram of hydrogen. Mixing that up is easy to do when under the pressure of taking a test, so the identifiers help avoid mistakes. The same is true of having the right number of significant digits in your given. When rushed and you working a problem, you might forget it was 3 SF and truncate your answer to 2 SF if you left the zero off and you just looked at the problem you set up. Students sometimes mix units up. In short, every step in this process is there to minimize the risk of making a clumsy error.

Once the "Given" and "Find" are listed, begin the problem by writing the "Find" followed by an equals sign:

$$mass(g)_{BS} =$$

then select one of the given pieces of information to the right of the equals sign. In the case of this problem, the choice is simple because there is only one given, but later problems may have 5 or more pieces of given information.

$$\text{mass}(g)_{BS} = (1.50\ \text{lb}_{BS})$$

At this point, you need information from beyond what is provided in the problem. In this case, it is a conversion factor. Some are in your textbook, others are available on the internet or other books. From your book, p. 26, there are 2.205 lbs in 1 kg. Converting this to a fraction and inserting it into the equation yields

$$\text{mass}(g)_{BS} = (1.50\ \text{lb}_{BS})\left(\frac{1\ \text{kg}_{BS}}{2.205\ \text{lb}_{BS}}\right)$$

when multiplying this, the units of lbs_{BS} divide away (cancel each other), leaving units of kilograms, but the problem asks for the mass in grams so another conversion factor is needed. Remember that conversion factors are typically exact numbers, so they usually don't affect the number of significant digits.

$$\text{mass}(g)_{BS} = (1.50\ \text{lb}_{BS})\left(\frac{1\ \text{kg}_{BS}}{2.205\ \text{lb}_{BS}}\right)\left(\frac{1000\ g_{BS}}{1\ \text{kg}_{BS}}\right)$$

Now, the kg_{BS}s divide out of the equation, leaving the requested units of grams (g). Doing the actual calculation provides the numerical part of the answer.

$$\text{mass}(g)_{BS} = (1.50\ \text{lb}_{BS})\left(\frac{1\ \text{kg}_{BS}}{2.205\ \text{lb}_{BS}}\right)\left(\frac{1000\ g_{BS}}{1\ \text{kg}_{BS}}\right) = 680\ g_{BS} = 6.8 \times 10^2\ g_{BS}$$

Finally, you should check your answer for reasonableness. For that you need to use the English-metric conversion factors that you were required to memorize. For example, in this case, 1.5 is about three-quarters (75%) of a kilogram and a kilogram is 1000 g. Therefore you expect an answer around 750 g. 680 is close enough to be considered "reasonable."

Most dimensional analysis problems are more complicated than this, typically because there will be two or more inputs (given information). In this case, it is necessary to pick one of the given as the first number entered into the calculation. There is no hard and fast rule that always works, but typically the answer will have units and if one of the given pieces of information has the same or similar unit (e.g. both are masses), choose the given with the same unit type. By unit type, both meters and kilometers count as a length unit. We will explore this further when we get to this type of problem.

2.7 Percent

Percentages are common way of expressing the amount of a material relative to the whole in chemistry. For example, if there are 2.5 g of sand in a mixture of salt and sand that weights 10.0 g, one quarter of the sample would be sand and so 25% of the sample would be sand. Mathematically, the calculation is

$$\%\text{sand} = \frac{2.5\ g_{sand}}{10.0\ g_{total}} \times 100\% = 25\%$$

so, in general,

$$\%\text{component} = \frac{\text{amount}_{component}}{\text{amount}_{total}} \times 100\%$$

An important aspect of percentages is that units matter, even though they cancel out. For example, rubbing alcohol is typically a solution of isopropyl alcohol in water. It is typically sold as a 70% v/v solution, which means that 70% of the volume is alcohol and 30% is water. If the same solution were labeled on a mass basis, the solution would be 65% alcohol by mass and 35% by water. Thus, it is important to be clear about quantities that are being used in the calculation. (It is true, that within a unit type, e.g. mass, as long as the units are the same, the volume is calculated. So, when calculating the volume percent of alcohol in this example, one could use either milliliters or quarts and still get the same 70%.)

2.8 Measurement of Temperature

As for metric vs. English measurement systems, there are two temperature scales used in everyday life. In this case, only 7 countries use Fahrenheit for temperatures, while the rest of the world uses Celsius. There is a third scale (Kelvin), closely related to Celsius, that is usually used in scientific calculations. Like the English scale, you will never use the Fahrenheit scale in scientific calculations. Unlike, the English scale, aside from temperatures between the freezing and boiling temperatures of water, being able to convert between the units isn't overly helpful.

The formula for interconverting °F and °C is

$$°C = \frac{°F - 32\ °F}{1.8\ °F/°C}$$

Three useful temperature comparison points that are useful to remember are:

1) melting point of water (ice) = 32 °F = 0 °C
2) boiling point of water = 212 °F = 100 °C
3) room temperature = 78 °F = 25 °C

"Room temperature" is not an official value. It represents a commonly used temperature, however.

The Kelvin scale is the same as the Celsius scale, but with 100 added to the values.

$$K = °C + 217.15$$

Note that the Kelvin scale does not use the degree (°) sign. This is because Fahrenheit and Celsius are relative scales, Kelvin is an absolute scale. For Celsius, the temperatures are relative to the boiling and melting points of pure water. In contrast, Kelvin temperatures are measured against and absolute zero point, just like mass (i.e. the mass of nothing is zero). In calculations, Kelvin temperatures are almost always used. The only time you will see Celsius used is when a temperature difference is needed. In this case, the difference in starting and ending temperature is the same on both scales because the individual "degrees" are the same.

2.9 Density

Density is the first situation we encounter with a property including multiple units. Density is the ratio of the mass of a substance to its volume ($\frac{\text{mass}}{\text{volume}}$) and for most solids and liquids has units of g/cm^3 or g/mL. For gases, volumes are typically measured in liters. Density allows us to discuss why solids can float on liquids and that a block of balsa wood feels lighter than an identical block of iron. Density is temperature dependent and so densities are usually reported with a temperature. This is because most substances expand when heated, but the mass remains constant. A result is that most densities decline as the substances warm.

Consider the water and isopropyl alcohol discussed in Section 2.7. Water has a density of 1.00 g/mL at 25 °C and isopropyl alcohol is 0.785 g/mL. Thus, if we have equal volumes of water and isopropyl alcohol, the water would weigh more. If we had equal masses, the isopropyl alcohol would occupy the larger volume. We can show this mathematically using dimensional analysis.

Assume 10.0 mL each of water and isopropyl alcohol, what are their respective masses:

$mass_{H2O} = (10.0\ mL_{H2O})(\frac{1.00\ g_{H2O}}{mL_{H2O}}) = 10.0\ g_{H2O}$

$mass_{IPA} = (10.0\ g_{H2O})(\frac{0.785\ g_{IPA}}{mL_{IPA}}) = 7.85\ g_{H2O}$

Next, assume 10.0 g each of water and isopropyl alcohol, what are their respective volumes:

$V_{H2O} = (10.0\ g_{H2O})(\frac{mL_{H2O}}{1.00\ g_{H2O}}) = 10.0\ mL_{H2O}$

$V_{IPA} = (10.0\ g_{IPA})(\frac{mL_{IPA}}{0785\ g_{IPA}}) = 12.7\ mL_{IPA}$

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